DOE Perspectives

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Experiment Strategies

The Discrete Factor Dilemma

Some industrial problems have discrete factors. Discrete factors have only certain levels that are possible — they are not continuous. An example of a two-level discrete factor is lot number for a part. You can use a part from lot A or lot B, but you have no other choice. Discrete factors require their own special designs and analysis.

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In this month's DOE Perspectives we will answer the question, "How can I study a two-level discrete factor with a quadratic model." It will also provide you with a Hardin-Sloane design for the quadratic model to tackle 2-level discrete factor problems that also have two process factors.

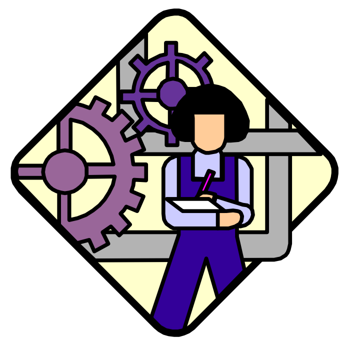
The Design

Table 1 (page 2) lists the runs for a 2-level discrete factor experiment with two process factors. The columns labeled D_1 and D_2 are the two levels of one discrete factor D. The columns labeled P_1 and P_2 are two process factors. The design has 12 runs. We recommend that you add enough replicates to get an s with 5 degrees of freedom. The design is I-optimal; it's the design with the minimum average variance of prediction. That may sound like Greek, but it means you will, on the average, have the smallest possible confidence limits on your predictions if you use this design.

This design is easy to use: if a 1 is in column D_1, you use that level for that run. If a 1 is in column D_2, you use that level. The process factors are coded on the -1 to 1 scale. To convert the design to the natural units (the units you will use to set the factor levels, i.e. pH, temperature, etc.) for your experiment, use the following formula:

$$X=(x(b-a)+a+b)/2$$

where: X is in the natural units
x is coded on the -1 to 1 scale
a is the lower limit of the factor in the natural units
b is the upper limit of the factor in the natural units



An Example

Let's look at an

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example that uses this design. You want to make an ink that can be made from either of 2 black dyes. Your company's manufacturing department will only use one dye, so mixing the dyes is out of the question. You want to determine which black dye, the amount of dye, and the amount of additive Z to use to make an ink giving the best print quality. Your upper limit for amount of dye is 7% and your lower limit is 0.5%. Your upper limit on additive Z is 30% and your lower limit is 10%. The bulk of your ink will be water.

First let's name the columns: D_1 is Dye A, D_2 is Dye B, P_1 is % Dye in the ink, and P_2 is % Additive Z in the ink. Now let's look at run number 3. You will use dye A (there is a 1 in column D_1), you will use 0.5% dye (there is a -1 in column P_2), and you will use 22% additive Z (there is a 0.2085 in column P_2, so

$$X=(0.2085(30-10)+10+30)/2 =22\%).$$

Each of the other runs is determined similarly.

Run Order	D1	D2	P1	P2
8	1	0	-1	-1
12	0	1	0	0
3	1	0	-1	0.2085
6	0	1	-1	1
4	1	0	-0.2085	1
1	0	1	0	0
11	1	0	1	-0.2085
7	0	1	-1	-1
10	1	0	1	1
2	0	1	1	1
5	1	0	0.2085	-1
9	0	1	1	-1

Table 1 — 2-level discrete factor with 2 process factors

Analysis

This design is analyzed differently than standard process designs. D_2 is actually not included in the model. If D_1 is 1, then D_2 must be 0 and vice versa. You also do not need a D_1^2 term in the model.

Defining the model is easy using STRATEGY[&]. First you highlight all of the factors in the factor column except D_1. Next you select the model type, "Full Quadratic," and the interaction order as "2." Finally, you click on the tab, "Suppress Terms," and highlight "D^2."

If you are using another software package you will need to fit your results to the following model:

$$Y=b_0+b_1x_1+b_2x_2+b_3x_3 +b_{12}x_1x_2+b_{13}x_1x_3 +b_{23}x_2x_3+x_2^2+x_3^2$$

Where: $x_1 = D_1, x_2 = P_1, x_3 = P_2$

When you make contour plots you must choose slices which make sense. You could make a plot for D1 = 0.5, but it would have no physical meaning since D1 can only be 1 or 0.

Searching for the optimum also requires care. If you are using STRATEGY, go to "Computing Sweet Spots" and click on the tab "Factor Limits." In the row labeled "Discrete?" click on the button in the column for D1. This tells STRATEGY that D1 is a discrete factor so it won't report impossible Sweet Spots.

If you are using another software, check its documentation to find out how to optimize discrete factors.

3- Discrete-Levels

The 3-discrete-level design with 2 process factors is also available for the quadratic model. It is a similar design, but it allows one extra level of the discrete factor (e.g. one more dye). Leave out column D_3 and the terms D_1^2, D_2^2, and D_1D_2 when you analyze your results. The model is

$$Y=b_0+b_1x_1+b_2x_2+b_3x_3+b_4x_4 +b_{13}x_1x_3+b_{14}x_1x_4 +b_{23}x_2x_3+b_{24}x_2x_4 +b_{34}x_3x_4+b_{33}x_3^2+b_{44}x_4^2$$

Where: $x_1 = D_1$, $x_2 = D_2$, $x_3 = P_1$, $x_4 = P_2$

If you would like the discrete factor designs discussed in this issue, please send E-mail to **info@ExperimentStrategies.com** with the subject "Send Discrete Designs."

[&]**STRATEGY** is available from Process Builder — (206)364-5740 www.ProcessBuilder.com